DIRECT CP VIOLATION IN NONLEPTONIC KAON DECAYS BY AN EFFECTIVE CHIRAL LAGRANGIAN APPROACH AT $O(p^6)$[1]

A.A. Bel'kov[1†], G. Bohm[2], A.V. Lanyov[1] and A.A. Moshkin[1]

(1) *Particle Physics Laboratory, Joint Institute for Nuclear Research,*
141980 Dubna, Moscow region, Russia
(2) *DESY–Zeuthen, Platanenallee 6, D–15735 Zeuthen, Germany*
† *E-mail: belkov@cv.jinr.dubna.su*

Abstract

A self-consistent analysis of $K \to 2\pi$ and $K \to 3\pi$ decays within a unique framework of chiral dynamics applied to the QCD-corrected weak nonleptonic quark lagrangian has been performed. The results on $K \to 2\pi$ amplitudes at $O(p^6)$, including the value for ε'/ε, are compared with the experiment to fix phenomenological B-factors for mesonic matrix elements of nonpenguin and penguin four-quark operators. The dependence of B-factors on different theoretical uncertainties and experimental errors of various input parameters is investigated. Finally, we present our estimates at $O(p^6)$ for the CP-asymmetry of linear slope parameters in the $K^\pm \to 3\pi$ Dalitz plot.

Keywords: direct CP violation, nonleptonic decays, K-mesons, chiral model

The starting point for most calculations of nonleptonic kaon decays is an effective weak lagrangian of the form [1, 2]

$$\mathcal{L}_w^q(|\Delta S| = 1) = \sqrt{2}\, G_F\, V_{ud} V_{us}^* \sum_i \widetilde{C}_i\, \mathcal{O}_i \,, \tag{1}$$

which can be derived with the help of the Wilson operator product expansion (OPE) from elementary quark processes, with additional gluon exchanges. In the framework of perturbative QCD the coefficients $\widetilde{C}_i$ are to be understood as scale and renormalization scheme dependent functions.

In (1), $\mathcal{O}_i$ are the four-quark operators, defined either by combinations of products of quark currents ($i = 1, 2, 3, 4$, non-penguin diagrams) or, in case of gluonic ($i = 5, 6$) and electro-weak ($i = 7, 8$) penguin operators, by products of quark densities. The operators $\mathcal{O}_i$ with $i = 1, 2, 3, 5, 6$ describe weak transitions with isospin change $\Delta I = 1/2$ while the operator $\mathcal{O}_4$ corresponds to $\Delta I = 3/2$ transition and operators $\mathcal{O}_{7,8}$ – to mixture of $\Delta I = 1/2$ and $\Delta I = 3/2$ amplitudes.

The general scheme of the meson matrix element calculation by using of the weak lagrangian (1) is based on the quark bosonization approach [3]. The bosonization procedure establishes the correspondence between quark and meson currents (densities) and product of currents (densities). Finally, it leads to the effective lagrangian for nonleptonic kaon decays in terms of bosonized (meson) currents and densities:

$$\bar{q}\gamma_\mu \frac{1}{4}(1 \mp \gamma^5)\lambda^a q \;\Rightarrow\; J_{L/R\,\mu}^{a\,(mes)}, \quad \bar{q}\frac{1}{4}(1 \mp \gamma^5)\lambda^a q \;\Rightarrow\; J_{L/R}^{a\,(mes)} \,.$$

---

[1]Talk at the XV Int. Seminar on High Energy Physics Problems "Relativistic Nuclear Physics & Quantum Chromodynamics", Dubna, September 25–29, 2000.

The meson currents/densities $J^a_{L/R\mu}$ and $J^a_{L/R}$ are obtained from the quark determinant by variation over additional external sources associated with the corresponding quark currents and densities [3]. From the momentum expansion of the quark determinant to $O(p^{2n})$ one can derive the strong lagrangian for mesons $\mathcal{L}_{eff}$ of the same order and the corresponding currents and densities $J^a_{L/R\mu}$ and $J^a_{L/R}$ to the order $O(p^{2n-1})$ and $O(p^{2n-2})$, respectively. Thus, the bosonization approach gives us the correspondence between power counting for the momentum expansion of the effective chiral lagrangian of strong meson interactions,

$$\mathcal{L}^{(mes)}_s = \mathcal{L}^{(p^2)}_s + \mathcal{L}^{(p^4)}_s + \mathcal{L}^{(p^6)}_s + \ldots,$$

and power counting for the meson currents and densities:

$$\mathcal{L}^{(p^n)}_s \;\Rightarrow\; J^{(p^{n-1})}_\mu \;(\text{currents})\,; \quad \mathcal{L}^{(p^n)}_s \;\Rightarrow\; J^{(p^{n-2})} \;(\text{densities})\,.$$

Some interesting observations on the difference of the momentum behavior of penguin and non-penguin operators can be drawn from power-counting arguments. The leading contributions to the vector currents and scalar densities are of $O(p^1)$ and $O(p^0)$, respectively. Since in our approach the non-penguin operators are constructed out of the products of currents $J^a_{L\mu}$, while the penguin operators are products of densities J^a_L, the lowest-order contributions of non-penguin and penguin operators are of $O(p^2)$ and $O(p^0)$, respectively. However, due to the well-known cancelation of the contribution of the gluonic penguin operator $\mathcal{O}_5$ at the lowest order [4], the leading gluonic penguin as well as non-penguin contributions start from $O(p^2)$ [2]. Consequently, in order to derive the currents which contribute to the non-penguin transition operators at the leading order, it is sufficient to use the terms of the quark determinant to $O(p^2)$ only. At the same time the terms of the quark determinant to $O(p^4)$ have to be kept for calculating the penguin contribution at $O(p^2)$, since it arises from the combination of densities, which are of $O(p^0)$ and $O(p^2)$, respectively. In this subtle way the difference in momentum behavior is revealed between matrix elements for these two types of weak transition operators; it manifests itself more drastically in higher-order lagrangians and currents.

This fact makes penguins especially sensitive to higher order effects. In particular, the difference in the momentum power counting behavior between penguin and non-penguin contributions to the isotopic amplitudes of $K \to 3\pi$ decays, which appears in higher orders of chiral theory, leads to the dynamical enhancement of the charge asymmetry of the Dalitz-plot linear slope parameter [5, 3].

In our approach the Wilson coefficients $\widetilde{C}_i$ in the effective weak lagrangian (1) are treated as the phenomenological parameters which should be fixed from the experiment. They are related with the Wilson coefficients $\widetilde{C}^{QCD}_i(\mu)$, calculated in perturbative QCD [6], via the $\widetilde{B}_i$-factors:

$$\widetilde{C}^{ph}_i = \widetilde{C}^{QCD}_i(\mu)\widetilde{B}_i(\mu)\,.$$

The coefficients $\widetilde{C}^{QCD}_i(\mu)$ contain a small imaginary parts and can be presented in the form of a sum of z and y components

$$\widetilde{C}^{QCD}_i(\mu) = \widetilde{C}^{(z)}_i(\mu) + \tau\,\widetilde{C}^{(y)}_i(\mu), \quad \tau = -\frac{V_{td}V^*_{ts}}{V_{ud}V^*_{us}}\,. \tag{2}$$

---

[2]There is no cancellation of the contribution of the electromagnetic penguin operator $\mathcal{O}_8$ at the lowest order and the leading contributions start in this case from $O(p^0)$

Respectively, the amplitudes of nonleptonic kaon decays also contain z- and y- components,

$$\mathcal{A} = \mathcal{A}^{(z)} + \tau \mathcal{A}^{(y)}, \tag{3}$$

which can be expressed in terms of z- and y-components of Wilson coefficients. The dominating contributions $\mathcal{A}^{(i)}$ of the four-quark operators $\mathcal{O}_i$ and $\tilde{B}_i$-factors may be written as

$$
\begin{aligned}
\mathcal{A}^{(z,y)} &= \left[-\tilde{C}_1^{(z,y)}(\mu) + \tilde{C}_2^{(z,y)}(\mu) + \tilde{C}_3^{(z,y)}(\mu) \right] \tilde{B}_1(\mu)\, \mathcal{A}^{(1)} \\
&\quad + \tilde{C}_4^{(z,y)}(\mu) \tilde{B}_4(\mu)\, \mathcal{A}^{(4)} + \tilde{C}_5^{(z,y)}(\mu) \tilde{B}_5(\mu)\, \mathcal{A}^{(5)} + \tilde{C}_8^{(z,y)}(\mu) \tilde{B}_8(\mu)\, \mathcal{A}^{(8)}.
\end{aligned}
$$

The observable effects of direct CP-violation in the nonleptonic kaon decays are caused by the y-components of their amplitudes (3). In particular, the ratio ε'/ε can be expressed as

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = \mathrm{Im}\,\lambda_t\,(P_0 - P_2), \quad P_I = \frac{\omega}{\sqrt{2}|\varepsilon||V_{ud}||V_{us}|} \frac{\mathcal{A}_I^{(y)}}{\mathcal{A}_I^{(z)}},$$

where $\mathrm{Im}\,\lambda_t = \mathrm{Im}\,V_{ts}^* V_{td} = |V_{ub}||V_{cb}|\sin\delta$; $\omega = \mathcal{A}_2^{(z)}/\mathcal{A}_0^{(z)}$, and $\mathcal{A}_I$ are isotopic amplitudes of $K \to 2\pi$ decay corresponding to the $\pi\pi$ final states with the isospin $I = 0, 2$.

There are the following theoretical uncertainties which appear both from short distance (Wilson coefficients) and long distance (effective chiral lagrangians and $\tilde{B}_i$-factors) contributions to the kaon decay amplitude:

- Dependence on the parameter $\mathrm{Im}\,\tau \sim \mathrm{Im}\,\lambda_t$ arising from the imaginary part of Wilson coefficient (2).

- Regularization scheme dependence which arises when calculating Wilson coefficients beyond the leading order (LO) of QCD in the next-to-leading orders: naive dimensional regularization (NDR), t'Hooft-Veltman regularization (HV).

- Dependence of Wilson coefficients on choice of the renormalization point μ and QCD scale $\Lambda_{\overline{MS}}^{(4)} = (325 \pm 110)$ MeV.

- Dependence of meson matrix elements on the structure constants L_2, L_3, L_4, L_5, L_8 of the general form of the effective chiral lagrangian introduced at $O(p^4)$ by Gasser and Leutwyler [7].

- Dependence of meson matrix elements on the structure constants of the effective chiral lagrangian at $O(p^6)$ [8].

- Factors $\tilde{B}_i$ ($i = 1, 4, 5, 8$) for dominating contributions of four-quark operators $\mathcal{O}_i$ to the meson matrix element (4).

- Dependence on choice of the regularization scheme to fix the UV divergences resulting from meson loops.

In the present paper we have combined a new systematic calculation of mesonic matrix elements for nonleptonic kaon decays from the effective chiral lagrangian approach with Wilson coefficients $\tilde{C}_i^{QCD}(\mu)$, derived by the Munich group [6] for $\mu = 1$ GeV and $m_t = 170$ GeV. For the parameter $\mathrm{Im}\,\lambda_t$ we have used the result obtained in [9]: $\mathrm{Im}\,\lambda_t = (1.33 \pm 0.14) \cdot 10^{-4}$. We performed a complete calculation of $K \to 2\pi$ and $K \to 3\pi$ amplitudes at

$O(p^6)$ including the tree level, one- and two-loop diagrams. For the structure coefficients L_i of the effective chiral lagrangian at $O(p^4)$ we used the values fixed in [10] from the phenomenological analysis of low-energy meson processes. The structure coefficients of the effective chiral lagrangian at $O(p^6)$ have been fixed theoretically from the modulus of the logarithm of the quark determinant of the NJL-type model (see [8] for more details). The superpropagator regularization has been applied to fix UV divergences in meson loops. The isotopic symmetry breaking (π^0-η-η' mixing) was taken into account at the tree level. In [11] one can find more technical details of the calculation of $K \to 2\pi$ amplitudes.

In our phenomenological analysis the results on the $K \to 2\pi$ amplitudes, including the value for ε'/ε, are compared with the experiment to fix the phenomenological $\widetilde{B}_i$-factors for the mesonic matrix elements of nonpenguin and penguin four-quark operators. As experimental input we used the experimental values of the isotopic amplitudes $\mathcal{A}_{0,2}^{(exp)}$ fixed from the widths of $K \to 2\pi$ decays and the world average value $\mathrm{Re}\,\varepsilon'/\varepsilon = (19.3\pm3.8)\times10^{-4}$ which includes both old results of NA31 [12] and E731 [13] experiments and recent results from KTeV [14] and NA48 [15]. The output parameters of the performed $K \to 2\pi$ analysis are the factors $\widetilde{B}_1$, $\widetilde{B}_4$ and $\widetilde{B}_5$ for a fixed value of $\widetilde{B}_8$.

The dependence of $\widetilde{B}_i$-factors on different theoretical uncertainties and experimental errors of various input parameters is investigated by applying the "Gaussian" method. Using Wilson coefficients derived in [6] in various regularization schemes (LO, NDR, HV) for different values of the QCD scale $\Lambda_{\overline{MS}}^{(4)}$, we calculated the probability density distributions for $\widetilde{B}_i$-factors obtained by using Gaussian distribution for all input parameters with their errors. As an example, the probability densities for the parameters $\widetilde{B}_1$, $\widetilde{B}_4$, $\widetilde{B}_5$ calculated with $\widetilde{B}_8 = 1$ and $\Lambda_{\overline{MS}}^{(4)} = 325$ MeV are shown in Figure 1. We have shown the necessity for a rather large gluonic penguin contribution to describe the recently confirmed large experimental ε' value (the factor $\widetilde{B}_5$ is found well above 1). Figure 2 shows the correlations between $\widetilde{B}_5$ and $\widetilde{B}_8$ calculated for central values of all input parameters. From this figure one can see that even for $\widetilde{B}_8 = 0$ values of $\widetilde{B}_5 > 2$ are necessary to explain the large value of ε'/ε. It should be emphasized, that for even larger values of $\widetilde{B}_5$, the contributions of nonpenguin operators to the $\Delta I = 1/2$ amplitude are still dominating (see Figure 3). The large $\widetilde{B}_1$ and $\widetilde{B}_5$ values may be a hint that the long-distance contributions, especially to $\Delta I = 1/2$ amplitudes, are still not completely understood. An analogous conclusion has been drawn in [9], where possible effects from physics beyond the Standard Model are also discussed.

Finally, we present our predictions for the CP-asymmetry of linear slope parameters in the $K^{\pm} \to 3\pi$ Dalitz plot. These predictions are based on a new calculation of $K \to 3\pi$ amplitudes at $O(p^6)$ within the same effective lagrangian approach. The obtained $K \to 3\pi$ amplitudes include the same theoretical uncertainties as in case of the $K \to 2\pi$ analysis. The values of $\widetilde{B}_1$, $\widetilde{B}_4$, $\widetilde{B}_5$ fixed from the $K \to 2\pi$ analysis are used as phenomenological input to the $K \to 3\pi$ estimates which have been performed in a self-consistent way by the Gaussian method.

The linear slope parameter g of the Dalitz plot for $K \to 3\pi$ decays is defined through the expansion of the decay probability over the kinematic variables

$$|T(K \to 3\pi)|^2 \; \propto \; 1 + gY + \dots$$

where Y is a Dalitz variable:

$$Y = \frac{s_3 - s_0}{m_\pi^2}, \quad s_3 = (p_K - p_{\pi_3})^2, \quad s_0 = \frac{m_K^2}{3} + m_\pi^2,$$
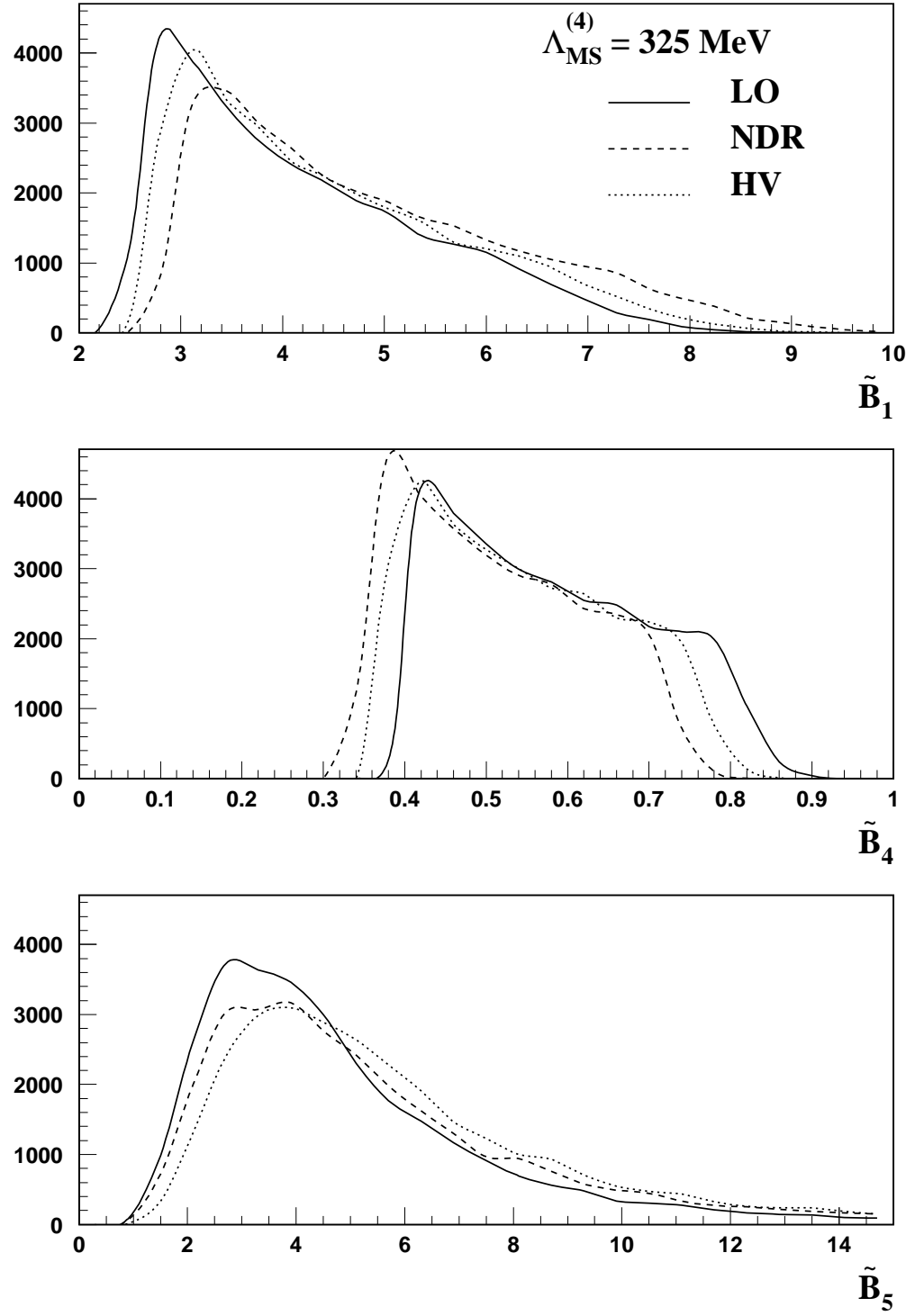
4

Figure 1: Probability density distributions for factors $\widetilde{B}_1$, $\widetilde{B}_4$ and $\widetilde{B}_5$ with $\widetilde{B}_8 = 1$.
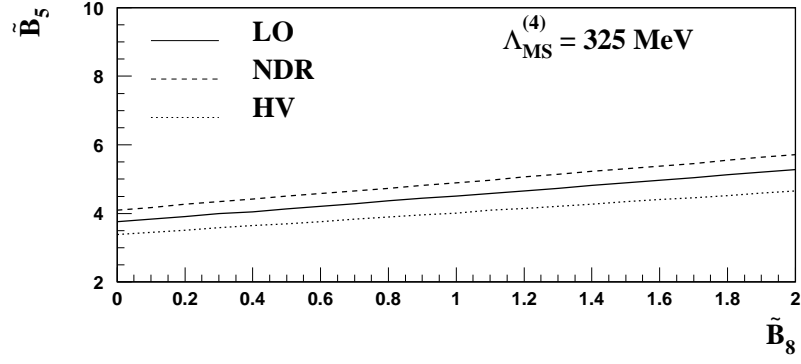
Figure 2: Correlations between parameters $\widetilde{B}_5$ and $\widetilde{B}_8 = 1$.
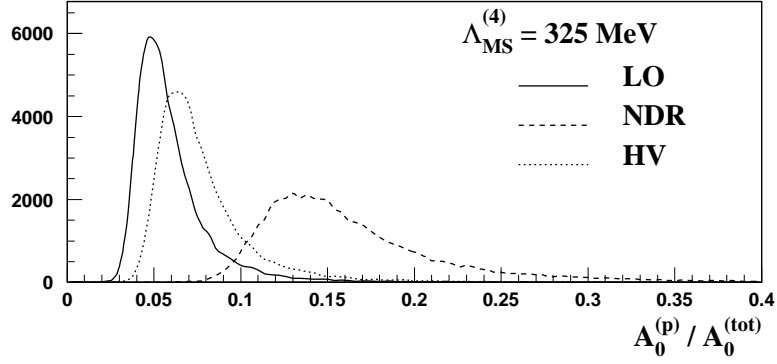

Figure 3: Probability density distributions for the relative contribution of penguin operators to the $\Delta I = 1/2$ amplitude.
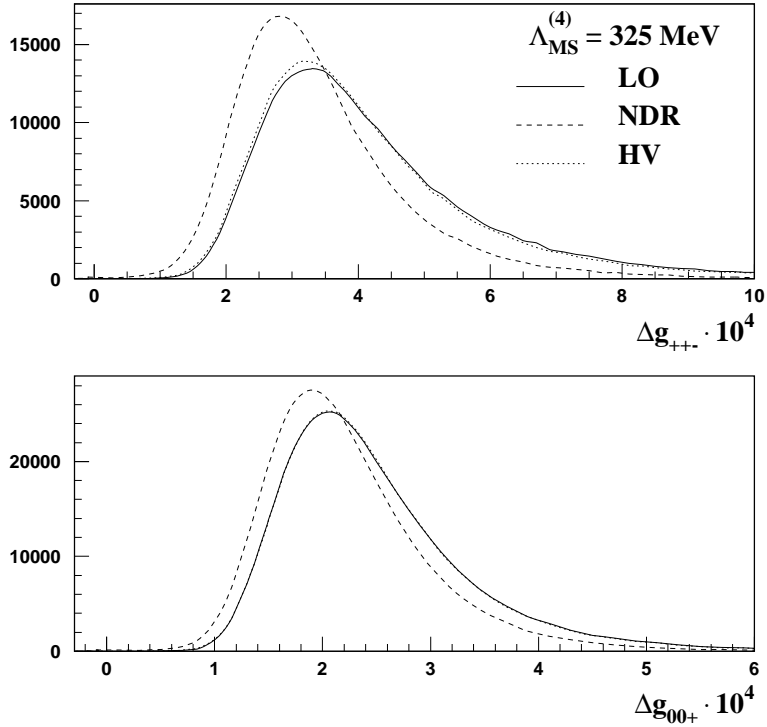

Figure 4: Probability density distributions for the CP-asymmetry of linear slope parameters of $K^\pm \to \pi^\pm\pi^\pm\pi^\mp$ and $K^\pm \to \pi^0\pi^0\pi^\pm$ decays.

and the notation π_3 belongs to the non-equivalent pion in the decays $K^\pm \to \pi^\pm\pi^\pm\pi^\mp$ and $K^\pm \to \pi^0\pi^0\pi^\pm$. Direct CP-violation leads to a charge asymmetry of the linear slope parameter,

$$\Delta g(K^\pm \to 3\pi) = \frac{g(K^+ \to 3\pi) - g(K^- \to 3\pi)}{g(K^+ \to 3\pi) + g(K^- \to 3\pi)}\,.$$

In Figure 4 we show the probability density distributions for $K^\pm \to \pi^\pm\pi^\pm\pi^\mp$ and $K^\pm \to \pi^0\pi^0\pi^\pm$ decays calculated with $\widetilde{B}_8 = 1$ and $\Lambda^{(4)}_{\overline{MS}} = 325$ MeV. Upper and low bounds for Δg_{++-} and Δg_{00+} for different values of $\Lambda^{(4)}_{\overline{MS}}$ in LO, NDR and HV regularization schemes ($\widetilde{B}_8 = 1$) obtained by the Gaussian method are shown in table 1. The limits without brackets correspond to the confidence level of 68% while the limits in brackets – to the confidence level of 95%. Summarizing these results, we have obtained the following upper and lower bounds for the charge symmetries of the linear slope parameter:

$$2.1 \,<\, \Delta g_{++-} \cdot 10^4 \,<\, 6.2\,, \quad 1.4 \,<\, \Delta g_{00+} \cdot 10^4 \,<\, 3.5 \ \text{ with CL=68\%;}$$

$$1.4 \,<\, \Delta g_{++-} \cdot 10^4 \,<\, 10.4\,, \quad 1.0 \,<\, \Delta g_{00+} \cdot 10^4 \,<\, 5.1 \ \text{ with CL=95\%.}$$

Table 1. Upper and low bounds for Δg_{++-} and Δg_{00+} (in units 10^{-4}).

Δg	$\Lambda^{(4)}_{\overline{MS}}$ (MeV)	LO		NDR		HV	
		min	max	min	max	min	max
Δg_{++-}	215	2.5	5.9	2.1	4.7	2.6	6.0
		(1.8	10.1)	(1.5	7.3)	(1.9	10.2)
	325	2.6	6.0	2.2	4.7	2.6	5.8
		(1.9	10.2)	(1.5	7.2)	(1.8	9.7)
	435	2.7	6.2	2.2	4.7	2.6	5.8
		(1.9	10.4)	(1.4	7.0)	(1.8	9.5)
Δg_{00+}	215	1.5	3.3	1.4	2.9	1.6	3.4
		(1.1	4.8)	(1.0	4.2)	(1.2	5.1)
	325	1.6	3.4	1.4	3.0	1.6	3.3
		(1.2	5.0)	(1.0	4.3)	(1.2	4.9)
	435	1.7	3.5	1.5	3.1	1.7	3.4
		(1.2	5.1)	(1.0	4.3)	(1.2	5.0)

With some experimental updates and theoretical refinements our new estimates confirm the dynamical enhancement mechanism for the charge asymmetry Δg by higher order contributions in the effective chiral lagrangian approach which was first observed in [5]. The predicted slope parameter asymmetry, although small, may be in reach of current high statistics experiments [16]. Already with lower statistics, new measurements of quadratic slope parameters of $K \to 3\pi$ decays, including the neutral channels, would lead to the improved theoretical understanding of the nonperturbative part of nonleptonic kaon decay dynamics.

References

[1] A.I. Vainshtein, V.I. Zakharov and M.A. Shifman, JETP **72**, 1275 (1977).

[2] F.J. Gilman and M.B. Wise, Phys. Rev. **D20**, 2392 (1979).

[3] A.A. Bel'kov, G. Bohm, A.V. Lanyov and A. Schaale, Phys. Part. Nucl. **26**, 239 (1995).

[4] R.S. Chivukula, J.M. Flynn and H. Georgi, Phys. Lett. **B171**, 453 (1986).

[5] A.A. Bel'kov, G. Bohm, D. Ebert and A.V. Lanyov, Phys. Lett. **220B**, 459 (1989).

[6] G. Buchalla, A.J. Buras and M.E. Lautenbacher, Rev. Mod. Phys. **68**, 1125 (1996).

[7] J. Gasser and H. Leutwyler, Nucl. Phys. **B250**, 465 (1985).

[8] A.A. Bel'kov and A.V. Lanyov, Phys. Part. Nucl. **29**, 33 (1998).

[9] S. Bosch, A.J. Buras, M. Gorbahn et al., Nucl. Phys. **B565**, 3 (2000).

[10] J. Bijnens, G. Ecker and J. Gasser, in The Second DAΦNE Physics Handbook, eds. L. Maiani, G. Pancheri, N. Paver, INFN, Frascati, 1995.

[11] A.A. Bel'kov, G. Bohm, A.V. Lanyov and A.A. Moshkin, Preprint JINR E2-99-236, Dubna, 1999; hep-ph/9907335.

[12] G.D. Barr et al., Phys. Lett. **B317**, 233 (1993).

[13] L.K. Gibbons et al., Phys. Rev. Lett. **70**, 1203 (1993).

[14] A. Alavi-Harati et al., Phys. Rev. Lett. **83**, 22 (1999).

[15] G. Barr [NA48 collaboration], in *Proc. of the 19th Intl. Symp. on Photon and Lepton Interactions at High Energy LP99* ed. J.A. Jaros and M.E. Peskin, eConf **C990809** (2000); hep-ex/9912042.

[16] R. Batley et al., CERN/SPSC 2000-003, CERN/SPSC/P253 add.3, Jan 2000, 25pp.